Confinement and the second vortex of the SU(4) gauge group

S. Deldar[†] and S. Rafibakhsh[‡]
Department of Physics, University of Tehran,
P.O. Box 14395/547, Tehran 1439955961, Iran
[†]*E-mail: sdeldar@ut.ac.ir*
[‡]*E-mail: rafibakhsh@ut.ac.ir*
(Dated: February 1, 2008)

We study the potential between static SU(4) sources using the Model of Thick Center Vortices. Such vortices are characterized by the center elements $z_1 = \mathrm{i}$ and $z_2 = z_1^2$. Fitting the ratios of string tensions to those obtained in Monte-Carlo calculations of lattice QCD we get $f_2 > f_1^2$, where f_n is the probability that a vortex of type n is piercing a plaquette. Because of $z_2 = z_1^2$ vortices of type two are overlapping vortices of type one. Therefore, $f_2 > f_1^2$ corresponds to the existence of an attractive force between vortices of type one.

I. INTRODUCTION

It is a well known fact that Quantum Chromodynamics (QCD) is truly described by the SU(3) gauge group. However, the mechanism of confinement is still under intensive discussion and is an attractive open subject. Studying the SU(N) gauge theories with N greater than three, helps us to better understand different aspects of QCD. Confinement implies that the potential between static sources has a linear term

$$V \simeq \sigma R \tag{1.1}$$

where σ is the string tension and R is the distance between the two sources. SU(N) gauge theories with $N > 3$ have some characteristics that SU(3) does not have. For example, there exists only one universal string tension for the SU(3) gauge theory but for SU(N) with $N > 3$, the number of independent stable string tensions are equal to $\mathrm{int}[\frac{N}{2}]$. The values of these string tensions will constrain the details of the confinement models. Both meta-stable string tensions which are the string tensions at intermediate distances and the stable string tensions which are the string tensions at large distances, are important, in this respect. Stable string tensions depend on the N-ality k of the corresponding representation. In fact, at large distances where the distance between two sources increases, a pair of gluons pops out of the vacuum and couples with the initial sources. They do not change the N-ality of the representation but reduce the dimension of the representation to the lowest dimension of the corresponding N-ality. Therefore, stable (asymptotic) string tensions depend on the N-ality of the representation. On the other hand, at intermediate distances, the string tensions depend on the dimension of the representation. For this region, the potential energy is not large enough to produce a pair of gluons and therefore, the dimension of the original sources does not change.

For stable strings, there are some existing theories describing the ratio of $\frac{\sigma_k}{\sigma_\mathrm{f}}$ where σ_f and σ_k are the string tensions for fundamental quarks and for quarks of representations with N-ality k, respectively. The linear potential between two static sources may be explained by forming a chromoelectric flux tube carrying charge in the center Z_N of the gauge group. The most trivial idea is that the total flux is carried by k independent fundamental tubes. Then

$$\sigma_k = \tilde{k}\sigma_\mathrm{f} \qquad ; \qquad \tilde{k} = \min\{k, N-k\} \tag{1.2}$$

Because of charge conjugation we get $\sigma_k = \sigma_{N-k}$. Thus, for the SU(3) gauge group, $\sigma_2 = \sigma_1$ and one universal string tension is obtained. If one wants to study theories with more than one string tension, one has to study SU(N) with $N > 3$. The asymptotic Casimir scaling is another theory which claims that [1]

$$\frac{\sigma_k}{\sigma_\mathrm{f}} = \frac{k(N-k)}{N-1} \tag{1.3}$$

Calculations in brane M-theory [2], predict Sine-law scaling

$$\frac{\sigma_k}{\sigma_\mathrm{f}} = \frac{\sin\frac{k\pi}{N}}{\sin\frac{\pi}{N}} \tag{1.4}$$

On the other hand, for intermediate distances, lattice calculations show that the string tensions are roughly proportional to the eigenvalues of the quadratic Casimir operators [3]. In ref. [4] this phenomenon was dubbed "Casimir scaling". The Casimir scaling regime is expected to extend roughly from the onset of confinement to the onset of

screening [5]. There is another argument about the linear part of the potential at intermediate distances which claims that the string tension in this region is proportional to the number of fundamental flux tubes embedded into the representation [6]. The fundamental flux or string is the one that connects a fundamental heavy quark with an anti-quark. This idea is called "flux tube counting". In general, the fundamental strings do not interact at very large N. Thus if one interprets the meta stable string tensions between the higher representation sources to be proportional to the number of flux tubes with $\frac{1}{N}$ corrections; then for large enough distances these meta-stable strings will decay into stable strings with given N-ality, whose tension is likely to be described by the Sine formula or by Casimir scaling in some approximation which is not yet known so far.

In this article we study the string tensions of SU(4) static sources, within the model of thick center vortices. In our previous calculations [7], we have shown that using only the first non-trivial type of the vortices of the SU(4) gauge group, two different asymptotic string tensions may be obtained, one for $k = 1$ and another one for $k = 2$. Their ratio approaches asymptotically $\frac{\sigma_{k=2}}{\sigma_{k=1}} \simeq 2$ in agreement with flux tube counting, see Eq. (1.2). If one wants to get other asymptotic string ratios, e.g. those corresponding to lattice calculations, one has to use both vortex types of the SU(4) gauge group. In the following sections, after giving a brief review of the role of vortices for the confinement of quarks, potentials between static sources are calculated using both types of SU(4) vortices. Because of the relation between the ratio of the stable string tension, $\frac{\sigma_k}{\sigma_\mathrm{f}}$, where σ_f is the fundamental string tension, and the probabilities f_1 and f_2 of piercing plaquettes by vortices of type one and two, one can determine the ratio of the probabilities, $\frac{f_2}{f_1}$, by fixing $\frac{\sigma_k}{\sigma_\mathrm{f}}$ from lattice calculations or from the theories related to Eqs. (1.2)-(1.4). Then, the induced potentials may be determined from the model. We show that the meta-stable string tension ratio $\frac{\sigma_r}{\sigma_\mathrm{f}}$ is almost independent of the ratio of the asymptotic string tensions, $\frac{\sigma_k}{\sigma_\mathrm{f}}$, where σ_r is the string tension of representation r at intermediate distances. Approximate agreement with both Casimir scaling and flux tube counting is observed for all potentials at intermediate distances. The effect of the second vortex type which modifies the concavity of the potentials and the type of the interaction between vortices are also discussed.

II. POTENTIALS FOR TWO TYPES OF THICK CENTER VORTICES

The vortex model of QCD assumes that the vacuum of quantum chromodynamic is filled with vortices of finite thickness which carry magnetic fluxes corresponding to the center of the gauge group. In order that vortices have finite energy per unit length, their gauge potential at large transverse distances must be a pure gauge. The non-trivial nature of gauge transformations for producing the gauge potentials causes that the vortex cores have non-zero energy and makes vortices topologically stable. The potential energy between static sources induced by the vortices is [5]

$$V(R) = -\sum_{x\in A} \ln\left\{1 - \sum_{n=1}^{N-1} f_n (1 - \mathrm{Re}\mathcal{G}_r[\vec{\alpha}^n_C(x)])\right\}. \tag{2.1}$$

x is the location of the center of the vortex and C indicates the Wilson loop. A is the minimal area of the Wilson loop and the sum over positions x runs over all plaquettes in the plane of the loop. f_n represents the probability that any given unit area is pierced by a vortex of type n. $\mathcal{G}_r$ which gives the information about the flux distribution and the contribution that a vortex with its center in a specific plaquette may have to the Wilson loop is given by

$$\mathcal{G}_r[\vec{\alpha}] = \frac{1}{d_r} \mathrm{Tr}\exp[\mathrm{i}\vec{\alpha}.\vec{H}^r], \tag{2.2}$$

where d_r is the dimension of the representation r and $\{H^r_i, i = 1, 2, ..., N-1\}$ are the generators in this representation spanning the Cartan subalgebra. The vector $\vec{\alpha}_c(x)$ depends on the fraction of the vortex core which is enclosed by the loop, thus on the color structure and the position of the vortex and the shape of the loop. For each SU(N) gauge group, there are $N-1$ types of vortices, corresponding to the non-trivial center elements z_n. Vortices of type n and $N-n$ differ in the direction of the magnetic flux only and have the same probability f_n. Thus

$$f_n = f_{N-n} \qquad \text{and} \qquad \mathcal{G}_r[\vec{\alpha}^n_C(x)] = \mathcal{G}^\star_r[\vec{\alpha}^{N-n}_C(x)]. \tag{2.3}$$

Hence, among the three vortices of the SU(4) gauge group, two of them may be considered to be the same. Therefore, f_1 is equal to f_3 and $\mathrm{Re}\mathcal{G}_r[\vec{\alpha}^{(1)}_C(x)] = \mathrm{Re}\mathcal{G}_r[\vec{\alpha}^{(3)}_C(x)]$, where the upper index (n) in $\vec{\alpha}^{(n)}_C(x)$ indicates the type of the vortex.

According to Eq. (2.1) the induced potential between SU(4) sources may be written as

$$V(R) = -\sum_x \ln\left\{1 - 2f_1\left(1 - \mathrm{Re}\mathcal{G}_r\left[\vec{\alpha}^{(1)}_C(x)\right]\right) - f_2\left(1 - \mathrm{Re}\mathcal{G}_r\left[\vec{\alpha}^{(2)}_C(x)\right]\right)\right\}. \tag{2.4}$$

Since $f_1 = f_3$, the first term in equation (2.4) is multiplied by 2. To determine $\mathcal{G}_r\,[\vec{\alpha}]$ from equation (2.2), H's and $\vec{\alpha}$'s for the SU(4) gauge group should be determined. The defining (fundamental) representation of the three generators H_i of the Cartan sub-algebra may be chosen as

$$\begin{aligned} H_1 &= \frac{1}{2}(1,-1,0,0), \\ H_2 &= \frac{1}{2\sqrt{3}}(1,1,-2,0), \\ H_3 &= \frac{1}{2\sqrt{6}}(1,1,1,-3). \end{aligned} \tag{2.5}$$

A center vortex completely linked to a Wilson loop, in the fundamental representation of the SU(N) gauge group, has the effect of multiplying the Wilson loop by a non-trivial center element $z_n = \exp\{\frac{2\mathrm{i}\pi n}{N}\}$

$$W(C) \to \exp\{\frac{2\mathrm{i}\pi n}{N}\}W(C) \qquad n = 1, 2, \cdots, N-1. \tag{2.6}$$

For the group SU(4), a vortex of type $n = 1$ requires $z_1 = \mathrm{i}$ and a vortex of type $n = 2$ requests $z_2 = -1$. The percentage of linking we describe by a function $\eta(x)$ which is zero when the vortex does not touch the loop and equal to one if the vortex is entirely contained within the loop. Then, the influence of a vortex of type n on the Wilson loop is

$$\mathcal{G}_r[\vec{\alpha}^{(n)}] = \frac{1}{4}\mathrm{Tr}\exp\{\mathrm{i}\vec{\alpha}^{(n)}(x)\vec{H}\}, \qquad \text{with} \qquad \vec{\alpha}^{(n)}(x) = \alpha^{(n)}_{\max}\vec{e}\,\eta(x) \tag{2.7}$$

A possible choice for the unit vector $\vec{e}$ and $\alpha^{(n)}_{\max}$ is

$$\vec{e} = (0,0,1), \quad \alpha^{(1)}_{\max} = \pi\sqrt{6}, \quad \alpha^{(2)}_{\max} = 2\pi\sqrt{6}, \tag{2.8}$$

leading to

$$\mathcal{G}_r[\vec{\alpha}^{(n)}] = \frac{1}{4}\mathrm{Tr}\exp\{\mathrm{i}\eta(x)n\pi\sqrt{6}H_3\} = \frac{1}{4}[3\exp\{\mathrm{i}\eta(x)n\frac{\pi}{2}\} + \exp\{\mathrm{i}\eta(x)n\frac{3\pi}{2}\}] \tag{2.9}$$

We can choose other unit vectors which lead to a permutation of the matrix elements of H_3 or to a sign change of H_3 and leave the sum in Eq. (2.9) unchanged, e.g.

$$\vec{e} = (\sqrt{\frac{2}{3}}, \frac{\sqrt{2}}{3}, \frac{1}{3}) \qquad \text{with} \qquad \vec{e}\vec{H} = \frac{1}{2\sqrt{6}}\mathrm{diag}(3,-1,-1,-1) \tag{2.10}$$

We have used the following profile, see ref. [5],

$$\eta(x) = \frac{1}{2}[1 - \tanh(ay(x) + \frac{b}{R})], \tag{2.11}$$

where

$$y(x) = \begin{cases} x - R & \text{for} \quad |R-x| \le |x| \\ -x & \text{for} \quad |R-x| > |x| \end{cases} \tag{2.12}$$

is the distance of the vortex center to the nearest timelike side of the loop. The parameter a is of the order of the inverse of the vortex core thickness [5] and the parameter b takes into account that vortices can not be completely linked to small Wilson loops.

Therefore, the vectors $\vec{\alpha}^{(n)}(x)$satisfy the following conditions

1. Vortices which pierce the plane far outside the loop do not affect the loop. It means for fixed R, as $x \to \infty$, $\alpha \to 0$.
2. If the vortex core is entirely contained within the Wilson loop, then for vortex type one, $|\vec{\alpha}^{(1)}| = \sqrt{6}\pi$ and for vortex type two, $|\vec{\alpha}^{(2)}| = 2\sqrt{6}\pi$.
3. As $R \to 0$ then $|\alpha^{(n)}| \to 0$.

So far, we have determined the group valued functions $\mathcal{G}_r\,[\vec{\alpha}]$ of equation (2.2). However, the two parameters f_1 and f_2 are not specified yet. In the next section, we show that the asymptotic string tensions depend on the 4-ality class k of the Wilson loop and deduce from the asymptotic string tension ratios the probability ratios $\frac{f_2}{f_1}$.

III. ASYMPTOTIC STRING TENSIONS

The asymptotic string tensions can be determined from very large Wilson loops. In this regime, we can neglect the finite thickness of vortices and assume that vortices piercing the minimal area of the loop would insert a center element somewhere in the product of link variables. For the SU(4) gauge group, there are four center elements

$$z_0 = 1 \qquad z_1 = \exp\left(\frac{\pi \mathrm{i}}{2}\right) \qquad z_2 = \exp(\pi \mathrm{i}) \qquad z_3 = z_1^* = \exp\left(\frac{3\pi \mathrm{i}}{2}\right). \tag{3.1}$$

Which center element contributes depends on the type n of the vortex and the N-ality k of the representation r of the loop. The Young-tableaux of the lowest representations of 4-ality k for SU(4) and their dimensions are depicted in Fig. 1. One can easily verify in this figure that the N-ality k modulo N is given by the number of quarks which is

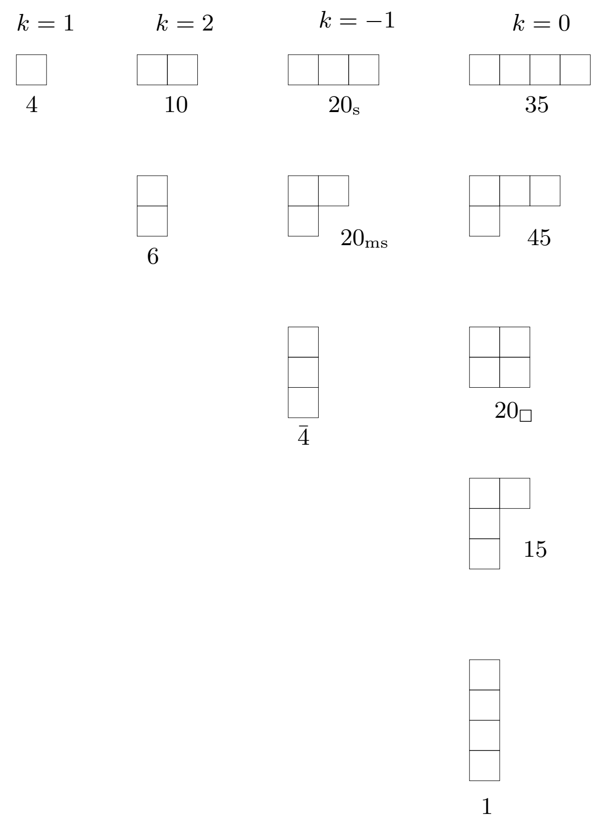

FIG. 1: The Young tableaus for SU(4) representations of various dimensions and their N-ality k is shown up to four-quark states.

equal to the number of squares in the Young tableau which is necessary to get such a representation. A representation r contributes with a factor $\mathcal{G}_r = z_n^{k(r)}$. Hence, from equation (2.1), the asymptotic potential for a representation r of N-ality k reads

$$V_r(R) \simeq -\sum_{x\in A} \ln\left\{1 - \sum_{n=1}^{N-1} f_n[1 - Re(z_n^{k(r)})]\right\} = \exp\{-A\sigma_r\}. \tag{3.2}$$

Using $f_1 = f_3$ from Eq. (2.3) we get for the fundamental representation of SU(4)

$$\begin{aligned} \sigma_{\mathrm{f}} &= -\ln\{1 - 2f_1\,[1 - \mathrm{Re}(z_1)] - f_2\,[1 - \mathrm{Re}(z_2)]\} = \\ &= -\ln[1 - 2f_1 - 2f_2] \simeq 2f_1 + 2f_2 \qquad \text{for} \qquad (f_1, f_2 \ll 1) \end{aligned} \tag{3.3}$$

For the diquark sources with 4-ality $k = 2$, like the sextet and the decuplet follows

$$\begin{aligned} \sigma_6 = \sigma_{10} &= -\ln\left\{1 - 2f_1\left[1 - \mathrm{Re}(z_1^2)\right] - f_2\left[1 - \mathrm{Re}(z_2^2)\right]\right\} = \\ &= -\ln(1 - 4f_1) \simeq 4f_1 \qquad \text{for} \qquad (f_1, f_2 \ll 1) \end{aligned} \tag{3.4}$$

Finally, we see that the asymptotic string tension of the 4-ality $k=0$ representations, which can be build by sources with the same numbers of quarks and antiquarks, is equal to zero. The most important member of this class is the adjoint representation

$$\sigma_{\mathrm{adj}} = -\ln\left\{1 - 2f_1\left[1 - \mathrm{Re}(z_1^0)\right] - f_2\left[1 - \mathrm{Re}(z_2^0)\right]\right\} = 0. \tag{3.5}$$

To summarize, the SU(4) asymptotic string tensions behave for small piercing probabilities like

$$\sigma_{\mathrm{f}} = 2f_1 + 2f_2, \qquad \sigma_6 = 4f_1, \qquad \sigma_{\mathrm{adj}} = 0. \tag{3.6}$$

This leads to the ratio

$$\frac{\sigma_6}{\sigma_{\mathrm{f}}} = \frac{2f_1}{f_1 + f_2} \tag{3.7}$$

of the asymptotic string tension for representations with 4-ality $k=2$ and 4-ality $k=1$.

This behavior that the asymptotic string tensions are equal for all quark sources of the same N-ality one can picture as pairs of gluons which pop out of the vacuum and screen the original sources. Therefore, the SU(4) gauge group has two universal asymptotic string tensions and Eq. (3.7) gives the ratio of these string tensions within the model of thick center vortices in terms of f_1 and f_2, of the probabilities of piercing plaquettes by vortices of type $n=1$ and $n=2$.

In the next section, we use the ratio of $\frac{\sigma_6}{\sigma_{\mathrm{f}}}$ from lattice calculations and from the predictions (1.2)-(1.4) to fix the ratio of $\frac{f_2}{f_1}$ by equation (3.7). Then the potentials between static sources are calculated and discussed.

IV. RESULTS AND DISCUSSION

In our previous calculations [7] for the gauge group SU(4), we assumed that vortices of type $n=1$ pierce plaquettes with the probability f_1 and no vortices of type $n=2$ are present, $f_2=0$. From Eq. (3.7) we get under these assumptions $\sigma_6 \simeq 2\sigma_{\mathrm{f}}$. This agrees with Eq. (1.2) and corresponds to the picture that a string of N-ality $k=2$ is built from two non-interacting fundamental strings. Lattice calculations do not confirm this scenario, since they predict

$$\begin{aligned} &\text{B. Lucini, } et.\ al \text{ [10]:} \quad \frac{\sigma_6}{\sigma_{\mathrm{f}}} \simeq 1.370(20) \quad \longrightarrow \quad \frac{f_2}{f_1} = 0.0460(7), \\ &\text{L. Del Debbio } et.\ al. \text{ [9]:} \quad \frac{\sigma_6}{\sigma_{\mathrm{f}}} \simeq 1.403(15) \quad \longrightarrow \quad \frac{f_2}{f_1} = 0.0426(5). \end{aligned} \tag{4.1}$$

On the other hand, the limit $f_1 = f_2$ leads to $\sigma_6 = \sigma_{\mathrm{f}}$, in obvious contradiction to the lattice results.

We fix now the ratio of $\frac{f_2}{f_1}$ to the values (4.1) predicted from the lattice data and adjust the absolute values of the probabilities f_1, f_2 a_1, a_2, b_1 and b_2 such that the potentials at intermediate distances become linear and get the best agreement with the intermediate string tensions of the lattice data. For both vortex types, the general form of the vortex profile introduced in equation (2.11) has been used. Fig. 2 shows the potentials of various representations versus R in the range of $R \in [1, 100]$, using

$$f_1 = 0.1, \quad f_2 = 0.046, \quad a_1 = 0.05, \quad b_1 = 4.0, \quad a_2 = 0.025, \quad b_2 = 8.0. \tag{4.2}$$

Indices one and two refer to vortices of type one and two. As one can see in Fig. 2 at large distances, zero 4-ality representations, like 15 (adjoint), $20_{\square}$ and 35 are screened. For non-zero 4-ality representations we get two different asymptotic string tensions. The potentials between the sources with dimension 20_s and 20_{ms} become parallel to that of the fundamental representation and the potentials of the diquark representations (6 and 10) get the same slope.

As expected, we find for all the potentials in Fig. 2 a linear region at intermediate distances. Table I shows the ratios $\frac{\sigma_r}{\sigma_{\mathrm{f}}}$ for the corresponding intermediate string tensions σ_r of some of these representations for $f_1 = 0.1$ and for some values of f_2. The errors in parentheses $\frac{\sigma_r}{\sigma_{\mathrm{f}}}$ show the systematic errors due to slight changes of the linear regime. The errors in square brackets are due to the uncertainties of the lattice data we have used. The first row contains the results of our previous work [7] with $f_2 = 0$, the second and the third line the above discussed results from the comparison with the lattice calculations of refs. [9] and [10] and the forth row the case $f_1 = f_2$. It is interesting to compare these values with the absence of vortices of type $n=1$ in the fifth row ($f_1 = 0$ and $f_2 = 0.1$) and with the fits for the asymptotic string tension to the predictions of the Casimir scaling law in Eq. (1.3) and the Sine law scaling in Eq. (1.4) in lines six and seven. It is clearly seen that slight changes of the piercing probabilities f_2 have only a

ratios	$\frac{\sigma_6}{\sigma_f}$	$\frac{\sigma_{15}}{\sigma_f}$	$\frac{\sigma_{10}}{\sigma_f}$	$\frac{\sigma_{20_s}}{\sigma_f}$	$\frac{\sigma_{35}}{\sigma_f}$	$\frac{\sigma_6}{\sigma_f}$(asymptotic)
$f_1 = 0.1,\ f_2 = 0$	1.50(5)	1.73(6)	1.88(5)	2.65(10)	3.21(27)	2
$f_1 = 0.1,\ f_2 = 0.0426$[5]	1.51(11)[2]	1.71(9)[2]	1.88(10)[2]	2.57(15)[3]	3.22(18)[3]	1.403[15]
$f_1 = 0.1,\ f_2 = 0.0460$[7]	1.50(8)[2]	1.69(14)[3]	1.86(8)[3]	2.52(10)[4]	3.16(14)[5]	1.370[20]
$f_1 = 0.1,\ f_2 = 0.1$	1.53(15)	1.71(16)	2.00(15)	2.72(21)	3.39(26)	1
$f_1 = 0,\quad f_2 = 0.1$	1.46(19)	1.48(9)	1.67(12)	2.18(17)	2.64(18)	0
$f_1 = 0.1,\ f_2 = 0.050$	1.43(20)	1.71(12)	1.88(13)	2.61(18)	3.22(23)	1.333
$f_1 = 0.1,\ f_2 = 0.042$	1.37(20)	1.78(18)	1.99(19)	2.71(26)	3.34(33)	1.414
Casimir ratio	1.33	2.13	2.4	4.2	6.4	
no. of fund. fluxes	2	2	2	3	4	

TABLE I: The string tension ratios $\frac{\sigma_r}{\sigma_f}$ for some SU(4) representations r at intermediate distances are shown in the first five columns and the values of $\frac{\sigma_6}{\sigma_f}$ in the asymptotic region in the last column. The first four rows treat the potentials for the piercing probabilities $f_1 = 0.1$ and increasing values of f_2, for our previous work [7] in the first row, for the fits to the lattice calculations of refs. [9] and[10] in the second and third row and for the case $f_1 = f_2$ in the forth row. Line five shows the ratios in the absence of vortices of type $n = 1$, line six fits to the predictions of the Casimir scaling law in Eq. (1.3) and line seven fits to the Sine law scaling in Eq. (1.4) in the asymptotic region. These values can be compared with the the ratio of eigenvalues $\frac{C_r}{C_f}$ of the quadratic Casimir operator i n line eight and with the number of fundamental fluxes in line nine. The errors in square brackets are due to the uncertainties of the lattice data we have used, the errors in parentheses $\frac{\sigma_r}{\sigma_f}$ show the systematic errors due to slight changes of the linear regime.

weak influence on the intermediate string tensions. This situation at intermediate distances differs drastically from the behavior at asymptotic distances, which was discussed above and is shown for comparison in the last column of Tab. I. The asymptotic string tensions of the lattice calculations in refs. [9] and [10] can be reproduced only with an appropriate density of vortices of type $n = 2$. The ratio of eigenvalues $\frac{C_r}{C_f}$ of the quadratic Casimir operator and finally the number of fundamental fluxes are represented in lines eight and nine. The lattice results in Table 17 of ref. [10] deviate for the representations 6 and 10 (there is no report on higher dimensional representations) by about 3.5% and 15% from Casimir scaling, our results in lines 2 and 3 of Table I by about 14% and 23%. The agreement with Casimir scaling gets worse as the dimensions of the representations increase. This fact is also observed in ref. [10] for the representations 6 and 10. Considering that the thick-center-vortices model does not reproduce the Coulombic part, the behavior of the linear part we get from the model is satisfying.

For values of f_1 and f_2 which are small enough compared to one, we can expand the logarithm in Eq. (2.4) around one and separate the contributions for the potentials produced by the two types of vortices. For our choice of the piercing probabilities in Fig. 2, $f_1 = 0.1$ and $f_2 = 0.046$, the violation of this additivity of the two contributions to the potential by the exact expression (2.4) is invisible, see Fig. 3, which compares the full decuplet potential with the potential produced by vortices of type $n = 1$ ($f_1 = 0.1$ and $f_2 = 0.0$) and vortices of type $n = 2$ ($f_1 = 0.0$ and $f_2 = 0.046$). It is nice to see that the vortices of type $n = 2$ produce $k = 2$ potentials which are asymptotically screened as predicted by Eq. (3.4). This can be understood by the QCD analogon of the Aharanov-Bohm effect. Carrying a two-quark source around a vortex corresponding to a $z_2 = z_1^2 = -1$ color magnetic flux leads to a phase of $z_2^2 = 1$, to a screened potential. It is easily understandable from Eq. (2.4) that the value of this screened potential is roughly proportional to the piercing probability f_2. Due to this screening effect, vortices of type $n = 2$ increase the string tension at intermediate distances only.

There is a probability f_1^2 that non-interacting vortices of type $n = 1$ have the same position and due to Eq. (2.8) are identified as vortices of type $n = 2$, $f_2 = f_1^2$. A comparison to the lattice calculations, see Eq. (4.1), leads to $f_2 > f_1^2$. This indicates an attraction between parallel vortex fluxes.

V. CONCLUSION

Confinement is one of the most interesting features of QCD which has been studied by both lattice gauge theory and phenomenological models. The model of thick center vortices is one of the phenomenological models which has been fairly successful to explain the linear part of the potentials. For the SU(N) gauge groups with $N \geq 4$ there exist vortices with different quantized fluxes. In this article we have studied the gauge group SU(4) which has two types of vortices, vortices of type $n = 1$ with magnetic flux corresponding to the first non-trivial center element $z_1 = \mathrm{i}$ of SU(4) and vortices of type $n = 2$ with a flux corresponding to $z_2 = -1$. We have shown that the ratio f_2/f_1 of

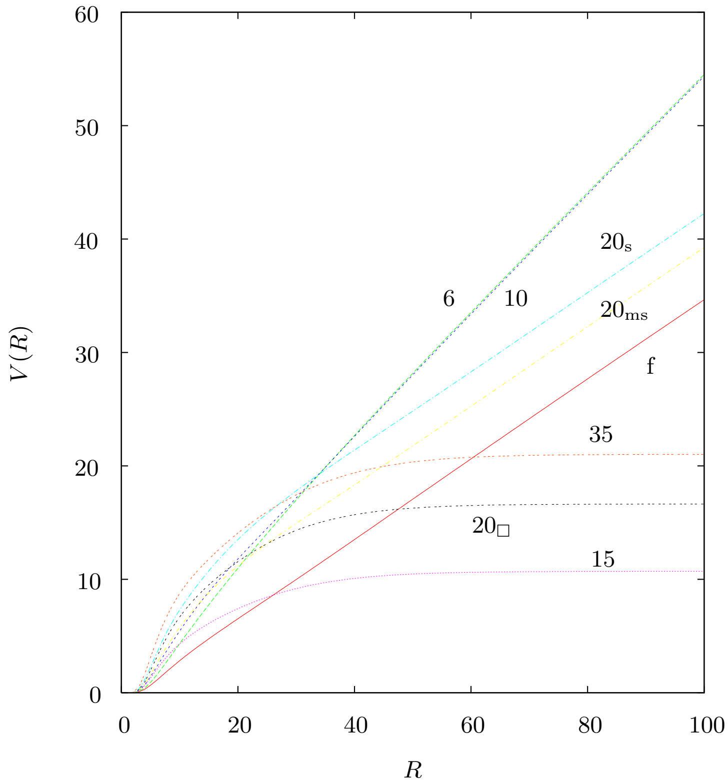

FIG. 2: Potentials for various representations of SU(4) using both types of vortices for piercing probabilities $f_1 = 0.1$ and $f_2 = 0.046$, see Eq. (4.1). The dimensions of the representations are indicated.

the probabilities for the piercing of plaquettes by vortices of both types determines the ratios of asymptotic string tensions of 4-alities $k = 0, 1$ and 2. We underline that the lattice results for the ratios of these string tension can be explained only by using both types of vortices . Adjusting these ratios to the slightly different results of refs. [9] and [10], the general features of the potentials and the string tensions at intermediate distances are yet indistinguishable.

Using vortices of type $n = 2$ only, the six and ten dimensional representations, which have 4-ality $k = 2$ and are possible two-quarks states, are screened at large distances. This results from the multiplication of the Wilson loop holonomy in a two-quark state by $z_2^2 = 1$. More generally, it follows that the $k = 2$ asymptotic string tension is independent of the probability f_2. The value of f_2 influences the $k = \pm 1$ asymptotic string tensions only.

Because of $z_2 = z_1^2$ a vortex of type $n = 2$ corresponds to two overlapping vortices of type $n = 1$. The analysis of the probabilities f_1 and f_2 gives an information about the interaction of vortices. Non-interacting vortices of type $n = 1$ are described by $f_2 = f_1^2$, whereas $f_2 > f_1^2$ indicates vortex attraction and $f_2 < f_1^2$ vortex repulsion. A comparison of our results with Monte-Carlo calculations indicates that vortices attract each other.

A consideration of vortices of type $n = 2$ modifies the concavity of the potentials which has been observed in previous calculations of SU(2), SU(3) and SU(4) potentials with only one type of vortices, see refs. [5], [7], [8] and which is not physical [11]. We conjecture that closed unquantised random magnetic flux lines of rather small size allow to remove this concavity and to introduce a Coulombic contribution with the correct sign.

VI. ACKNOWLEDGMENTS

We are deeply grateful to numerous discussions with M. Faber. We would also like to thank Š. Olejník, J. Greensite and M. Shifman for their valuable comments. This work is partly supported by the research council of the University of Tehran and partly by the grant "Structure of matter" awarded by the center of excellence to the Department of

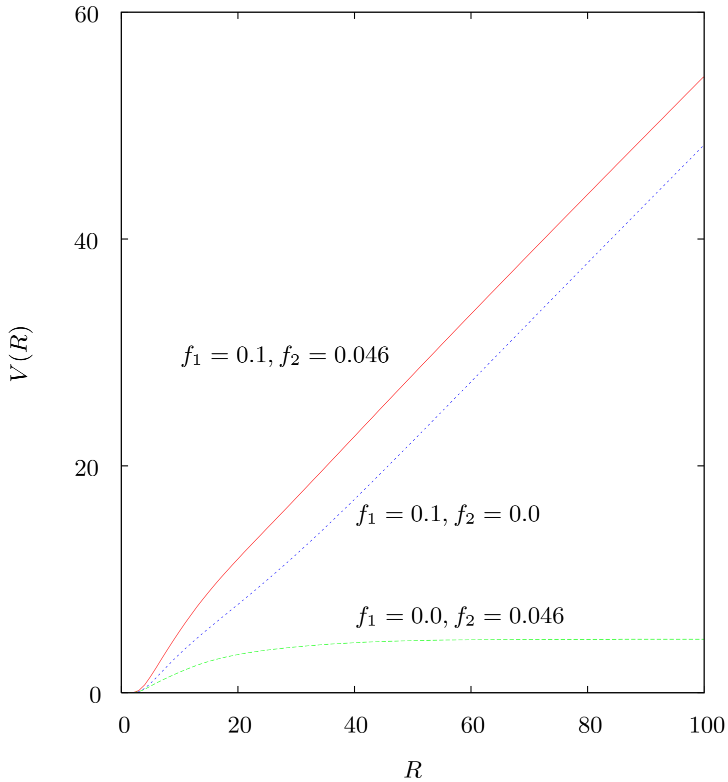

FIG. 3: For piercing probabilities f_1 and f_2 small compared to one the contributions of the two vortex types of SU(4) are almost completely additive as one can see in these decuplet potentials for the indicated piercing probabilities. Remarkable is the screening of potentials of 4-ality $k = 2$ produced by vortices of type $n = 2$ ($f_1 = 0.0$ and $f_2 = 0.046$) which is understandable by a QCD-Aharanov-Bohm effect.

Physics of the University of Tehran.

[1] J. Ambjorn, P. Olesen and C. Peterson, Nucl. Phys. B **240** (1984) 533; Nucl. Phys. B **244** (1984) 262; Phys. Lett. B **142** (1984) 410.

[2] M. R. Douglas and S. H. Shenker, Nucl. Phys. B **447** (1995) 271 [arXiv:hep-th/9503163]; A. Hanany, M. J. Strassler and A. Zaffaroni, Nucl. Phys. B **513** (1998) 87 [arXiv:hep-th/9707244].

[3] S. Deldar, Phys. Rev. D **62** (2000) 034509 [arXiv:hep-lat/9911008]; G. S. Bali, Phys. Rev. D **62** (2000) 114503 [arXiv:hep-lat/0006022].

[4] L. Del Debbio, M. Faber, J. Greensite and Š. Olejník, Phys. Rev. D **53** (1996) 5891 [arXiv:hep-lat/9510028].

[5] M. Faber, J. Greensite and Š. Olejník, Phys. Rev. D **57** (1998) 2603 [arXiv:hep-lat/9710039].

[6] C. Michael, arXiv:hep-ph/9809211; G. S. Bali, Phys. Rept. **343** (2001) 1 [arXiv:hep-ph/0001312]; A. Armoni and M. Shifman, Nucl. Phys. B **671** (2003) 67 [arXiv:hep-th/0307020].

[7] S. Deldar and S. Rafibakhsh, Eur. Phys. J. C **42** (2005) 319 [arXiv:hep-ph/0411184].

[8] S. Deldar, JHEP **0101** (2001) 013 [arXiv:hep-ph/9912428].

[9] L. Del Debbio, H. Panagopoulos, P. Rossi and E. Vicari, JHEP **0201**, 435 (2002) 009 [arXiv:hep-th/0111090].

[10] B. Lucini and M. Teper and U. Wenger JHEP **0406** (2004) 012 [arXiv:hep-lat/0404008].

[11] C. Bachas, Phys. Rev. D **33** (1986) 2723.